Optimal Coordination Policy for a Simple Supply Chain System with Economies of Scale in a JIT Environment

Tien-Yu Lin

Department of Marketing and Distribution Management, The Overseas Chinese Institute of Technology

100, Chiao Kwang Rd. Taichung 407, Taiwan

admtyl@ocit.edu.tw

***Abstract*— Recently several quantitative results have been published on the coordination policy between supplier and buyer, in which the critical issue is to determine join economic order quantities and shipments. Most of the previous studies on this issue assumed that the direct cost of the product is irrelevant; however, the direct cost of the product becomes a critical factor when economies of scale are involved in the supply chain system. Unlike traditional models in SCM field, this research proposes a new mathematical model in which the supplier enjoys the benefit of economies of scale in a JIT environment and offers the price to the buyer by cost-markup pricing. The objective of this paper is to determine the optimal number of shipments, the lot size of each shipment, and production/procurement cost in order to minimize the joint annual total cost incurred by both vendor and buyer. An efficient algorithm is developed to determine the optimal solution. Also, numerical example is given to illustrate the proposed model and algorithm .**

***Keywords*— Supply chain; Coordination; JIT; Economies of scale; lot-splitting shipment.**

1. INTRODUCTION

In recent years, the coordination issue in the supply chain management (SCM) has received considerable attention from academicians and practitioners. Numerous studies have demonstrated that substantial benefits can be achieved from coordination. In principle, coordination and the related issues at the interfaces of SCM can be grouped as four categories: procurement and production coordination, production and inventory coordination, production and distribution coordination, and distribution and inventory coordination [1]. One of the most interesting topics in production and inventory coordination is integrated decision making. Several researchers have examined many theoretical, as well as practical issues involving buyer-supplier coordination, as a means of attaining successful implementation of just-in-time (JIT)-based decision systems in an effort to minimize the supply costs. Thus, it is not uncommon to develop inter-linked, coordinated procurement, production and delivery schedules throughout the chain, with the goal of enhancing the performance of the entire supply chain, via joint optimization, rather than focusing on one or another individual party's performance objectives [2]. This implies that as optimal contract quantities and as optimal number of delivers must be determine at the outset of the contract. This contract further based on their integrated total cost function rather than buyer's and vendor's individual cost functions. Chung and Wee [3] also pointed out that a key technique of successful SCM is the application of JIT multiple shipment; increase of quality, productivity, and efficiency can be achieved through JIT multiple delivery agreement.

The idea of integrating the buyer's and the supplier's total cost functions was initialed by Goyal [4] in which he assumed that manufacturer does not produce item and purchases item from another supplier. Banerjee [5] considered a joint economic-lot-size model where question of pricing and lot-sizing decisions are settled through negotiations Goyal [6] further relaxed the lot-for-lot production assumption in Banerjee's model and assumed that whole lot is produced before the first shipment is made to the buyer. He also suggested the economic production quantity of vendor could be as integer multiple of the buyer's purchase quantity. Goyal

[7] and Hill [8] later proposed different delivery policies and suggested that successive shipments in a batch are inflated by a constant factor-(production rate/demand rate). The same idea further has been extended by Ha and Kim [9] who developed an integrated lot-splitting model of facilitating multiple delivers in small lot size in a relatively simple JIT environment. Hoque and Goyal [10] suggested an optimal procedure to a single-vendor, single-buyer production and inventory problem with both equal and unequal sized shipments, in which capacity constraint of transportation equipments was included. Pan and Yang [11] proposed an integrated inventory model with controllable lead time and with normally distributed demand. Israel and Moshe [12] identified the degree of independence and level of flexibility in terms of lot sizing and delivery scheduling in a single-vendor single–buyer system. Further, Huang [13] developed an optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. More recently, Chen and Kang [14] developed the integrated vendor-buyer cooperative inventory models with the permissible delay in payments to determine the optimal replenishment time interval and replenishment frequency. Simultaneously, Chan and Kingsman [15] developed a coordinated single-vendor multi-buyer supply chain model by synchronizing delivery and production in which the synchronization is achieved by scheduling the actual delivery days of the buyers and coordinating them with the vendor's production cycle. Banerjee et al. [2] proposed a mathematical model for coordinating the replenishment decisions for procurement, production and distribution inventories, associated with a single product multi-echelon supply chain environment. The general result of the above papers is that integrated lot sizing models reduce the total system costs; however, whilst they reduce the cost to the vendor they increase the costs to the buyer(Chan and Kingsman [15]). Attention has been thus put on examining mechanisms for the supplier to share the savings by offering quantity discounts to encourage the buyer to purchase larger quantities (Chan and Kingsman [15]; Weng [16]).

Monahan [17] was among the first researchers focusing on the study of quantity discounts for supply chain coordination, in which he assumed a lot-for-lot replenishment policy for a vendor and showed that a vendor could encourage the buyer to order large quantities by offering a price discount. Rosenblat and Lee [18] relaxed the lot-for-lot assumption in Monahan's model and allowed the vendor to purchase inter multiple of buyer's order quantity. Goyal [19] provided another model to determine the economic order policy under the amount of quantity discount offered by vendor. Recent studied have begun to consider how to realize supply chain corporation by using quantity discounts. Weng [16] developed a model to show that quantity discounts can promote the volume of demand, in which demand is the function of selling price, and achieve the joint profit maximization. Further, Weng [20] considered both all-unit and incremental quantity discount policy under single-vendor single-buyer supply system, where the buyer determined the selling price charged to customers. Chen et al. [21] considered single-supplier multiple-buyer distribution system in which a discount scheme is designed to achieve the integrated channel coordination. Later, Klastorin et al. [22] tested the issue of order coordination between a supplier and multipliers in a decentralized multi-echelon inventory/ distribution system in which a manufacturer offers a price discount to retailers when they coordinate the timing of their orders with the manufacturer's order cycle. Recently, Li and Liu [23] adopted quantity discounts as a tool to coordinate a two-echelon system with stochastic demand. Qin et al. [24] further employed the mechanism of quantity discounts and franchise fees to harmonize a two-echelon channel with a price-sensitive demand. Tsai [25] proposed a SCM model capable of treating various quantity discount functions in which the SCM model can be approximated to a linear mixed 0-1 programming problem solvable to obtain a effectively globally optimal solution. Shin and Benton [26] developed a quantity discount model that resolved the practical challenges associated with implementing quantity discount policies for supply chain coordination between a supplier and a buyer. More recently, Zhou et al. [27] proposed a quantity discount coordination schedule to coordinate the channel of a decentralized two-echelon supply chain and considered a current-stock-depend demand rate . It should be noted that all these works assumed that the supplier offered discounts to buyer. However, they neglected the effect of economics of scale for manufacturer while he/she simultaneously offers the quantity discounts to the buyer.

Based on the arguments above, unlike traditional models in SCM field, this research proposes a new mathematical model in which the supplier enjoys the benefit of economies of scale in a JIT environment and offers the price to the buyer by cost-markup pricing. That is, the manufacturer/supplier enjoys the benefit of economies of scale while the buyer has the benefit of quantity discounts. Further, this model considers a simple and practical scenario in which the shipment quantities to the buyer at each cycle are identical. The annual joint total cost function is developed and an efficient algorithm is established to find out the optimal solution. A numerical example is also given to illustrate the propose model and algorithm.

2. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are made throughout this paper to develop the mathematical model.

Notations:

D annual demand rate for buyer

F fixed transportation cost per trip

K ordering cost for buyer per order

N number of shipments per batch cycle, a positive integer

P annual production rate for supplier, $P > D$

Q order or production lot size in units

R receiving cost for buyer per trip

S vendor's setup cost per setup

I_b annual buyer's inventory carrying charge, expressed as a fraction of dollar value

I_v annual supplier's inventory carrying charge, expressed as a fraction of dollar value

c_j unit manufacturing cost of Qth level for supplier

r unit cost-markup charge for buyer, expressed as a fraction of dollar value. That is, we employee $(1+r)\, c_j$ as the unit purchasing cost for buyer.

Assumptions:

(1) the supply chain consists of a single supplier and a single buyer for each item
(2) demand for the item is constant over time
(3) production rate is uniform and finite
(4) products are shipped in a fixed quantity at a regular interval
(5) shortages are not allowed
(6) supplier enjoys the benefits of economics of scale and thus offers the all-unit quantity discounts to its buyer by employing cost-markup pricing. That is, the supplier provides the quantity discounts, which depends on the its economics of scale, to encourage the buyer ordering large quantities. The cost scheme is listed as Table1 in which c_j be the unit manufacturing cost of Qth level. That is, if $Q_{j-1} \le Q < Q_j$, then the unit manufacturing cost is c_j

Table 1. Manufacturing cost structure under economics of scale for supplier

j	$Q_{j-1} \le Q < Q_j$	c
1	$0 < Q < Q_1$	c_1
2	$Q_1 \le Q < Q_2$	c_2
3	$Q_2 \le Q < Q_3$	c_3
.	.	.
.	.	.
.	.	.
m	$Q_{m-1} \le Q < \infty$	c_m

3. MATHEMATICAL MODEL

Fig. 1 depicts the behavior of inventory level for the supplier, the buyer, and the system. Since the unit manufacturing cost for supplier, from this figure we have the supplier's annual holding cost as follows:

$$HC_{vendor} = \frac{c_j I_v \left[bold\ area - shaded\ area \right]}{Q/D}$$

$$= \frac{c_j I_v}{Q/D} \left\{ \left[Q \left(\frac{Q}{NP} + \frac{(N-1)Q}{ND} \right) - \frac{Q^2}{2P} \right] \right\}$$

$$- \frac{c_j I_v}{Q/D} \left\{ \frac{Q/D}{N} \left[\frac{Q}{N} + \frac{2Q}{N} + \dots + \left(\frac{N-1}{N} \right) Q \right] \right\}$$

$$= \frac{c_j I_v Q}{2N} \left(\frac{2D}{P} - \frac{ND}{P} + N - 1 \right)$$

Define $TC_j(N,Q)_{vendor}$ as the supplier's total cost per cycle if we assume the unit manufacturing cost, c_j, is valid for all Q. It is the sum of the manufacturing cost, setup cost, shipment cost, and inventory cost. Therefore, the total cost for supplier is given by

$$TC_j(N,Q)_{vendor} = c_j D + \frac{DS}{Q} + \frac{DNF}{Q} + \frac{c_j I_v Q}{2N} \left(\frac{2D}{P} - \frac{ND}{P} + N - 1 \right) \quad j = 1,2,\dots,m \qquad (1)$$

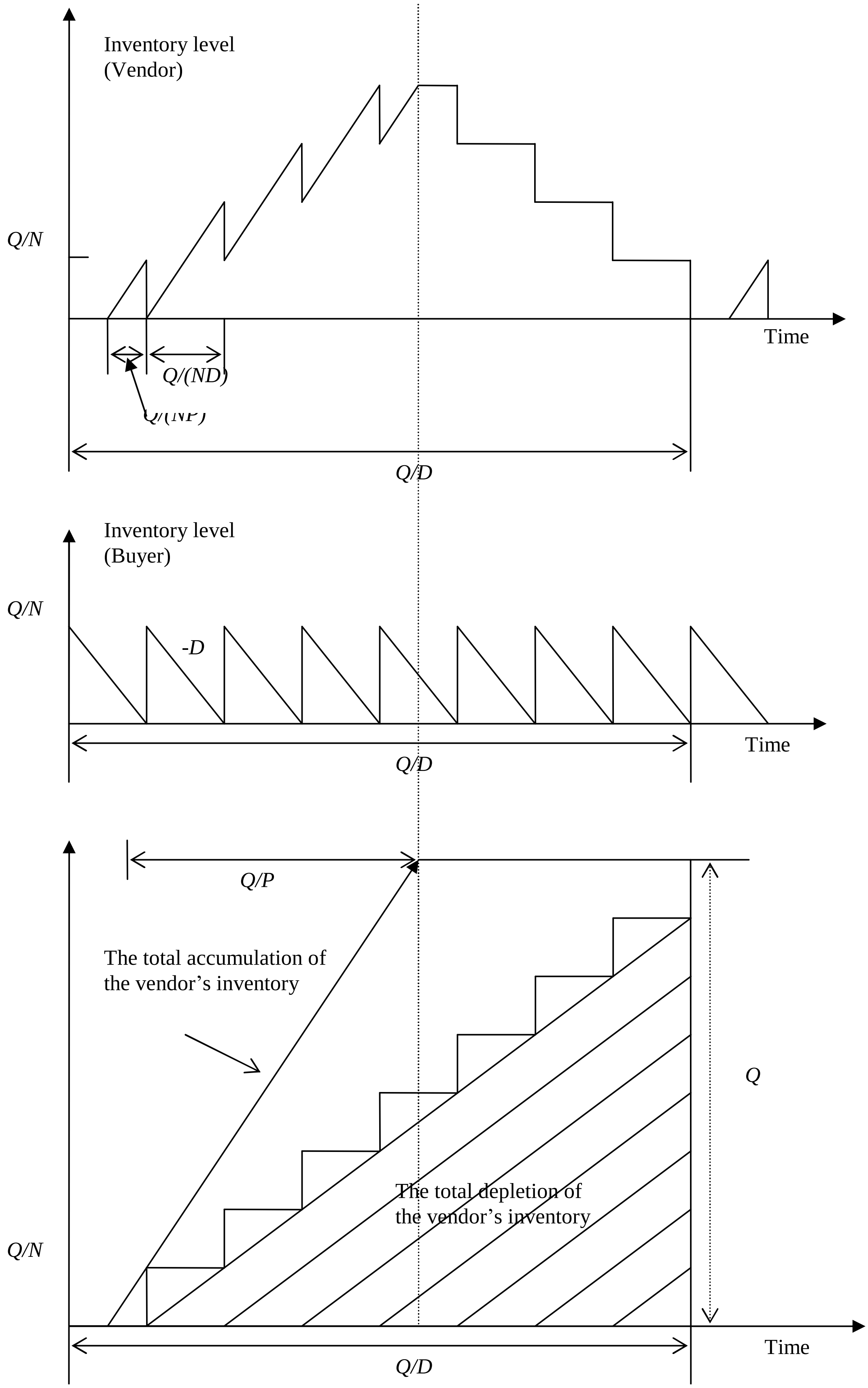

Fig. 1 Behavior of the inventory level for the vendor, the buyer, and the system

Further, let $TC_j(N,Q)_{buyer}$ denotes the buyer's total cost per cycle if we assume the unit purchasing price, $(1+r)c_j$, is valid for all *Q*. The buyer's total cost per cycle is the sum of purchasing cost, ordering cost, receiving cost and inventory cost. Thus,

$$TC_j(N,Q)_{buyer} = (1+r)c_j D + \frac{DK}{Q} + \frac{DNR}{Q} + \frac{c_j I_b Q}{2N},\ j = 1,2,...,\ m \qquad (2)$$

Note that the first term of summing Eqs. (1) and (2) yields joint annual total, $TC_j(N,Q)_{total}$, for both the supplier and the buyer as follows:

$$TC_j(N,Q)_{total} = c_j D(2+r) + \frac{D}{Q}(K+S+NR+NF) + \frac{c_j Q}{2N}\left\{(1+r)I_v + I_b\left(\frac{2D}{P} - \frac{ND}{P} + N - 1\right)\right\}$$
$$j = 1,2,...,m \qquad (3)$$

4. ALGORITHM

Our objective is to minimize the integrated total cost. To convenient analysis, we temporarily relax the number of deliveries *N* as continuous variable. Thus, it can easily be shown that the Hession matrix of Eq. (3) is positive definite. This provides that the joint annual total cost in Eq. (3) is convex. By taking the first derivatives of Eq. (3) with respect to *N* and *Q*, setting them equal to zero, and solving for *N* and *Q* simultaneously, one has

$$\tilde{N} = \sqrt{\frac{(K+S)[P((1+r)I_b + I_v) + 2DI_v]}{(R+F)I_v(P-D)}} \qquad (4)$$

and

$$Q_j^* = \sqrt{\frac{2D(K+S)}{c_j I_v(1-D/P)}},\ j = 1,2,...,m \qquad (5)$$

where $\tilde{N}$ is a possible number of deliveries and Q_j^* is the lowest point of manufacturing/ ordering quantities for each cost curve of c_j

Note that the possible number of deliveries in Eq. (4) is independent on the unit manufacturing/purchasing cost. Thus one can determine the possible number of shipments without considering the scheme of economics of scale or quantity discounts. Since the value of *N* is a positive integer, we let $\bar{N} = \lfloor \tilde{N} \rfloor$ in which $\lfloor \tilde{N} \rfloor$ is the greatest integer $< \tilde{N}$. Therefore, there exists one of two feasible shipments, $\bar{N}$ or $\bar{N}+1$, which minimize Eq. (3). To find out which number of shipments is optimal, we should make further analysis

Furthermore, the optimal production or order lot size can not be obtained by Eq. (5). This is because *Q* is the function of unit manufacturing/ purchasing cost for supplier/ buyer. Therefore, to obtain the optimal lot size, unit manufacturing/purchasing cost and number of shipments, an efficient algorithm, similar to Goyal [28], is developed as below:

Step 1. Obtain c_{N^Δ} as follows:

$$c_{N^\Delta} = \frac{1}{D(2+r)}\left\{c_m D(2+r) + \left(\frac{D}{Q_{m-1}} - \frac{D}{Q_m^*}\right)[K+S+N^\Delta(F+R)] - \frac{c_m(Q_{m-1} - Q_m^*)}{2N^\Delta} * \left\{(1+r)I_v + I_b\left(\frac{2D}{P} - \frac{N^\Delta D}{P} + N^\Delta - 1\right)\right\}\right.$$

where $Q_m^* = \sqrt{\frac{2D(K+S)}{c_m I_v(1-D/P)}}$ and

$$N^\Delta \in \{\bar{N}, \bar{N}+1\}$$

Step 2. For each manufacturing/purchasing cost, $c_m \le c_j < \min(c_{\bar{N}}, c_{\bar{N}+1})$, the possible optimal number of shipments, N_j^*, is determined by $\min(c_{\bar{N}}, c_{\bar{N}+1})$.

Step 3. Substitute $Q(c_j) = \max(Q_{j-1}, Q_j^*)$ and N_j^* into Eq. (3), determine the annual total cost $TC(N_j^*, Q(c_j))_{total}$

Step 4. Compare all the annual total cost obtained in step 3. The lowest annual total cost provides the optimal order quantity, $Q(c_j)$, and the optimal number of shipment, N^*

5. NUMERICAL EXAMPLE

In this section, we use the following parameters to illustrate the effectiveness of our model and algorithm developed in the previous section:

Production rate $P = 12{,}000$ units/year,
Demand rate $D = 10{,}000$ units/year,
Ordering cost for buyer $K = \$100$ /cycle,
Setup cost for supplier $S = \$200$ /cycle,
Transportation cost $F = \$120$ /delivery,
Receiving cost $R = \$50$ /delivery,
Unit cost-markup charge $r = 0.25$
Buyer's inventory carrying charge $I_b = 0.2$
Supplier's inventory carrying charge $I_v = 0.1$

In addition, the supplier enjoys the economics of scale and has the cost structure as Table 2.

Table 2. Manufacturing cost structure for supplier in the numerical example

j	$Q_{j-1} \sim Q_j$	c
1	$0 < Q < 1250$	$c_1 = 24$
2	$1250 \le Q < 2500$	$c_2 = 23$
3	$2500 \le Q < 3750$	$c_3 = 22$
4	$3750 \le Q < 5000$	$c_4 = 21$
5	$Q \ge 5000$	$c_5 = 20$

Considering the type of cost structure and using the algorithm developed in section 3, we can obtain the optimal manufacturing/ordering quantity, number of shipment and minimum annual total cost.

Step 1.

$$Q_5^* = \sqrt{\frac{2*10000*(100+200)}{20*0.1*(1-10000/12000)}} = 4242.64$$

$$\tilde{N} = \sqrt{\frac{(100+200)[12000*((1+0.25)*0.2+0.1)+2*10000*0.1]}{(50+120)*0.1*(12000-10000)}} = 5.79$$

$$\overline{N} = \lfloor 5.79 \rfloor = 5$$

$$c_{\overline{N}} = \frac{1}{10000*(2+0.25)}\{2*10000*(2+0.25)$$
$$+\left(\frac{10000}{5000}-\frac{1000}{4242.64}\right)[100+200+5*(120+50)]$$
$$-\frac{20*(5000-4242.64)}{2*5}*$$
$$\left\{(1+0.25)*0.1+0.2*\left(\frac{2*10000}{12000}-\frac{5*10000}{12000}+5-1\right)\right\}$$
$$=20.0087$$

$$c_{\overline{N}+1} = \frac{1}{10000*(2+0.25)}\{20*10000*(2+0.25)$$
$$+\left(\frac{10000}{5000}-\frac{1000}{4242.64}\right)[100+200+6*(120+50)]$$
$$-\frac{20*(5000-4242.64)}{2*6}*$$
$$\left\{(1+0.25)*0.1+0.2*\left(\frac{2*10000}{12000}-\frac{5*10000}{12000}+6-1\right)\right\}$$
$$=20.0024$$

Step 2. Considering the condition of manufacturing/purchasing cost break segment, there is only one case, $c_5 = 20$, satisfying $20 \le c_j < \min(20.0087, 20.0024)$. We also know the above minimum value, 20.0024, occurs at $N^* = \overline{N} + 1 = 6$.

Step 3.

The order lot size under $c_5 = 20$ is given by

$$Q(c_5 = 20) = \max[Q_{5-1}, Q_5^*] = \max[5000, 4242.64]$$
$$= 5000$$

Substitute $Q(c_5) = 5000$ and $N^* = 6$ into Eq. (3) under $c_5 = 20$, the annual total cost is $TC(6{,}5000(c_5 = 20))_{total} = 456112.2$.

Step 4. Based on the above case satisfying the relevant conditions, we can obtain the optimal order/ production lot size, $Q = 5000$, the manufacturing/purchasing cost, $c_5 = 20$, the number of shipments, $N^* = 6$, and the optimal annual total cost, $TC(6{,}5000(c_5 = 20))_{total} = 456112.2$

6. CONCLUSIONS

In this paper, we propose a new generalized production-inventory model in which the supplier enjoys the benefit of economies of scale in a JIT environment and offers the price to the buyer by cost-markup pricing. This model further considers a simple and practical scenario in which the shipment quantities to the buyer at each cycle are identical. The annual joint total cost function is developed and an efficient algorithm is established to find out the optimal order/production lot size, the manufacturing/ purchasing cost, the number of shipments, and the optimal annual total cost. Numerical example shows that our model is reasonable and our algorithm is simple and efficient.

The proposed models can be further enriched by incorporating more realistic assumption, such as probability demand, deteriorating item, finite rate of replenishment, and time value of money.

REFERENCES

[1] A.K. Arshinder and S.G. Deshmukh, "Supply chain coordination: Perspectives, empirical studies and research directions," *International Journal of Production Economics*, vol. 115, pp. 316-335, 2008.

[2] A. Banerjee, S.L. Kim, and J. Burton, "Supply chain coordination through effective multi-stage inventory linkages in a JIT environment," *International Journal of Production Economics*, vol. 108, pp. 271-280, 2007.

[3] C.J. Chung and H.M. Wee, "Optimizing the economic lot size of a three-stage supply chain with backordering derived without derivatives," *European Journal of Operational Research*, vol. 183, pp. 933-943, 2007.

[4] S.K. Goyal, "An integrated inventory model for a single supplier-single customer problem," *International Journal of Production Research*, vol. 15, pp. 107-111, 1976.

[5] A. Banerjee, "A joint economic-lot-size model for purchaser and vendor," *Decision Sciences*, vol. 17, pp. 292–311, 1986.

[6] S.K. Goyal, "A joint economic-lot-size model for purchaser and vendor: A comment," *Decision Sciences*, vol. 19, pp. 236–241, 1988.

[7] S.K. Goyal, "A one-vendor multi-buyer integrated inventory model: A comment," *European Journal of Operational Research*, vol. 82, pp. 209–210, 1995.

[8] R.M. Hill, "The single-vendor single-buyer integrated production–inventory model with a generalized policy," *European Journal of Operational Research*, vol. 97, pp. 493–499, 1997.

[9] D. Ha and S.L. Kim, "Implementation of JIT purchasing: An integrated approach," *Production Planning & Control*, vol. 8, pp. 152–157, 1997.

[10] M.A. Hoque and S.K. Goyal, "An optimal policy for a single-vendor single-buyer integrated production–inventory system with capacity constraint of the transport equipment," *International Journal of Production Economics*, vol. 65, pp. 305–315, 2000.

[11] J.C.H. Pan and J.S. Yang, "A study of an integrated inventory with controllable lead time," *International Journal of Production Research*, vol. 40, pp. 1263-1273, 2002.

[12] D. Israel and E.C. Moshe, "How far should JIT vendor–buyer relationships go?" *International Journal of Production Economics*, vol. 81-82, pp. 361–368, 2003.

[13] C. K. Huang, "An optimal policy for a single-vendor single buyer integrated production-inventory problem with process unreliability consideration," *International Journal of Production Economics*, vol. 91, pp. 91-98, 2004.

[14] L.H. Chen and F.S. Kang, "Integrated vendor-buyer cooperative inventory models with variant permissible delay in payments," *European Journal of Operational Research*, vol. 183, pp. 658–673, 2007.

[15] C.K. Chan and B.G. Kingsman, "Coordination in a single-vendor multi-buyer supply chain by synchronizing delivery and production cycles," *Transportation Research Part E: Logistics and Transportation Review*, vol. 43, pp. 90-111, 2007.

[16] Z.K. Weng, "Modeling quantity discounts under general price-sensitive demand functions: Optimal policies and relationships," *European Journal of Operational Research*, vol. 86, pp. 300–314, 1995.

[17] J.P. Monahan, "A quantity discount pricing model to increase vendor profits," *Management Science*, vol. 30, pp. 720-726, 1984.

[18] M.J. Rosenblat and H.L. Lee, "Economic production cycles with imperfect production processes," *IIE Transactions*, vol. 18, pp. 48–55, 1986.

[19] S.K. Goyal, "Comment on: A Generalized Quantity Discount Pricing Model to

Increase Supplier's Profits," *Management Science*, vol. 33, pp. 1635-1639, 1987.

[20] Z.K. Weng, "Channel coordination and quantity discounts," *Management Science*, vol. 41, pp. 1509-1522, 1995.

[21] F. Chen, A. Federgruen, and Y.S. Zheng, "Coordination mechanisms for a distribution system with one supplier and multiple retailers," *Management Science*, vol. 47, pp. 693-708, 2001.

[22] T.D. Klastorin, K. Moinzadeh, and J. Son, "Coordinating orders in supply chains through price discounts," *IIE Transactions*, vol. 34, pp. 679-589, 2002.

[23] J. Li and L. Liu, "Supply chain coordination with quatity discount policy," *International Journal of Production Economics*, vol. 101, pp. 89-98, 2006.

[24] Y. Qin, H. Tang, and C. Guo, "Channel coordination and volume discounts with price-sensitive demand," *International Journal of Production Economics*, vol. 105, pp. 43-53, 2007.

[25] J.F. Tsai, "An optimization approach for supply chain management models with quantity discount policy," *European Journal of Operational Research*, vol. 177, pp. 982-994, 2007.

[26] H. Shin and W.C. Benton, "A quantity discount approach to supply chain coordination," *European Journal of Operational Research*, vol. 180, pp. 601-616, 2007.

[27] Y.W. Zhou, J. Min, and S.K. Goyal, "Supply-chain coordination under an inventory- level-dependent demand rate," *International Journal of Production Economics*, vol. 113, pp. 518-527, 2008.

[28] S.K. Goyal, "A simple procedure for price break models," *Production Planning and Control*, vol. 6, pp.584-585, 1995.