An Adaptive Scaling Scheme to Normalize Output Information for Max-Log-MAP Convolutional Turbo Decoding

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***Abstract*— Convolutional turbo decoder with the Max-Log-MAP algorithm is well compromise between coding gain performance and decoding complexity. It has been shown that the decoding quality is improved by using a scaling factor within the extrinsic information as the number of iterations increases. This paper proposes a simple scaling technique to normalize the output information that extends the existing likelihood-decision-aided (LDA) stopping criterion. The proposal method is to count the number of sign differences in log-likelihood-ratio (LLR) information between two consecutive decoders, and then adaptively determines the corresponding scaling factor for each data block. Simulations show that it performs better in terms of the coding gain performance and the average number of iterations, as well as reducing the decoding power consumption and time latency.**

***Keywords*— Likelihood-decision-aided (LDA), coding gain improvement, scaling of output information, Max-Log-MAP, iterative turbo decoder.**

1. INTRODUCTION

In the mobile communication system, channel error control coding is incorporated into the transmission systems that are easily affected to noise and interference, with the purpose of achieving highly reliable communication at rates approaching channel capacity. Turbo codes were presented by Berrou, Glavieux, and Thitimajshimal [1] in 1993. In recent years, considerable interest has been devoted a great deal of attention because of their outstanding error performance and moderate decoding complexity. It was demonstrated that can achieve performance close to the Shannon limit. Turbo coding has attracted great interest in the last few years because of large coding gains. It has been chosen for the third mobile commu-ications standard (IMT-2000/3GPP) [5].

For turbo decoding algorithms, the Max-log-MAP algorithm has good compromise between performance and complexity [2]. Compared to nearly optimal Log-MAP algorithm for decoding complexity is fewer about half for finding the correction factors $f_c(x)$ which is a Jacobian term, however, it always suffers a performance degradation of about 0.5 dB [3]. Assume that additive white Gaussian noisy (AWGN) channels, theoretically, it is not necessary to estimate the SNR, because the Max-Log-MAP decoder is SNR independent [4].

In 2000, Pyndiah [6] highlighted that the standard deviation of the extrinsic information is very high in the first decoding steps and decreases as we iterate the turbo decoding. To take above fact into account, it was suggested to multiply the extrinsic information at the output of each turbo decoder by a fixed scaling factor $\alpha^{(i)}$. The evolution of $\alpha^{(i)}$ with the decoding iteration number i is

$$\alpha^{(i)} = [0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.0,\ 1.0,\ 1.0]. \quad (1)$$

This scaling factor $\alpha^{(i)}$ is also used to reduce the effect of the extrinsic information in the turbo decoder in the first decoding steps when the bit error rate (BER) is relatively high. It takes a small value in the first decoding steps and increases as the BER tends to zero.

In this paper, the scaling factor evaluated in [6] depends on the heuristic procedure instead of using the intuitive approach that the extrinsic information at the output of each soft-input soft-output (SISO) turbo decoder is normalized by an efficient scaling scheme that extends the likelihood-decision-aided (LDA) stopping criterion [8]. Our proposal scheme counts the number of sign changes in log-likelihood-ratio

(LLR) information between two consecutive decoders, and then adaptively determines the corresponding scaling factor for each frame. In Section II, we first present the proposed soft-input scaling scheme for turbo decoding. Then, Section III describes the system and simulations we conducted. The simulation results are also discussed in this section. Finally, conclusions are drawn in Section IV.

2. THE PROPOSED SCALING SCHEME

For simplicity, we consider a turbo code that consists of two identical RSC codes with feedback. Let $\{u_k,\ 1 \le k \le N\}$ be the information block of length N at time k, and $(x_k^s,\ x_k^p)$ are the systematic and the parity outputs of the turbo encoder for the static transition from the state $S_{k-1} = s'$ at time $k-1$ moves to state $S_k = s$, whose code bits are transmitted by BPSK modulated $\{0 \to -1, 1 \to +1\}$, and assume through a zero-mean additive white Gaussian noisy (AWGN) channel with a double-sided power spectral density of $N_0/2$.

The turbo decoding process is performed in an iterative manner. SISO decoding of the convolutional subcodes is done with the use of *a-priori* information of previous decoding steps. Here only relevant formulas of the Max-Log-MAP algorithm used are given. At the decoder module, two SISO decoders SISO1 and SISO2 are employed to produce the estimates $\{\hat{u}_k,\ 1 \le k \le N\}$. In the i th iteration, the first SISO decoder receives the channel sequence $(L_c \cdot y_{k,1}^s,\ L_c \cdot y_{k,1}^p)$ from the first encoder where y^s and y^p are the received channel codeword, and the a-priori information $L_{a,1}^{(i)}(u_k)$ provided by de-interleaving the extrinsic information $L_{e,2}^{(i-1)}(u_k)$ of the second SISO decoder in the $(i-1)$ th iteration. To simplify the calculation, the Max-Log-MAP algorithm applies the approximation

$$\ln \sum_i e^{X_i} \simeq \max_i \{X_i\}. \quad (2)$$

Thus, the branch metric are independent of the channel SNR, then can be omitted the effect of channel reliability L_c as follows

$$\Gamma_{k,1}^{(i)}(s',s) = \frac{1}{2}[x_{k,1}^s \cdot L_{a,1}^{(i)}(u_k) + x_{k,1}^s \cdot y_{k,1}^s + x_{k,1}^p \cdot y_{k,1}^p] \quad (3)$$

The forward and backward path metrics $A(s)$, $B(s)$ are defined as follows

$$A_{k,1}^{(i)}(s) = \max_{s'} \left\{ A_{k-1,1}^{(i)}(s) + \Gamma_{k,1}^{(i)}(s',s) \right\} \quad (4)$$

$$B_{k-1,1}^{(i)}(s) = \max_{s'} \left\{ B_{k,1}^{(i)}(s) + \Gamma_{k,1}^{(i)}(s',s) \right\}. \quad (5)$$

It can produce an improved *a-posteriori* information

$$L_1^{(i)}(u_k) = \max_{(s',s),u_k=+1} \{A_{k-1,1}^{(i)}(s') + \Gamma_{k,1}^{(i)}(s',s) + B_{k,1}^{(i)}(s)\}$$
$$- \max_{(s',s),u_k=-1} \{A_{k-1,1}^{(i)}(s') + \Gamma_{k,1}^{(i)}(s',s) + B_{k,1}^{(i)}(s)\}. \quad (6)$$

From $L_1^{(i)}(u_k)$ the extrinsic information can be derived which is used as the *a-priori* information in the second SISO decoder iteration

$$L_{e,1}^{(i)}(\hat{u}_k) = L_1^{(i)}(\hat{u}_k) - y_{k,1}^s - L_{a,1}^{(i)}(\hat{u}_k). \quad (7)$$

As described in [6] that the extrinsic information should be multiplied by a scaling factor at every iteration step to avoid increasing BER at low signal-to-noise ratios (SNRs). Based on this fact, the new scaling scheme extends the existing Likelihood-Decision-Aided (LDA) technique [8] to normal the associated soft output of a turbo decoder. The proposed adaptive scaling scheme is to compute the values of Δ function of the k th estimated bit $\hat{u}_k$ at the i th iteration for the decoders LDA1.

$$\Delta_1^{(i)}(\hat{u}_k) = \begin{cases} \frac{1}{N} & if\ \overline{L}_2^{(i-1)}(\hat{u}_k) \cdot \overline{L}_1^{(i)}(\hat{u}_k) > 0 \\ 0 & elsewhere \end{cases} \quad (8)$$

where N is the block length. Then, the scaling factors $\alpha_1^{(i)}$ associated with the block decoded by the decoders SISO1 at the i th iteration can be formulated as

$$\alpha_1^{(i)} = \sum_{k=1}^{N} \Delta_1^{(i)}(\hat{u}_k) \quad (9)$$

with the initial condition $\alpha_1^{(0)} = 0.5$. As the number of iterations increases, it is known that the LDA value of a block will tend toward 0 [8], oppositely the Δ value of an entire block will tend toward to 1.0. Therefore, the scaling factors $\alpha_1^{(i)}$ can be treated as a reliability indicator for a block to be decoded, and simultaneously offered a simple and effective rule for stopping the iterative decoding process when the scaling factor value of $\alpha_1^{(i)}$ is equal to 1.0.

Next, the second SISO decoder comes into operation. It uses the interleaved channel sequence $(L_c \cdot y_{k2}^s,\ L_c \cdot y_{k2}^p)$ from the second encoder and the a-priori information $L_{a,2}^{(i)}(u_k)$ derived by interleaving the extrinsic information

$L_{e,1}^{(i)}(u_k)$ of the first SISO decoder, and incorporating the proposed adaptive scaling factor $\alpha_1^{(i)}$ as follows

$$L_{a,2}^{(i)}(\hat{u}_k) = \alpha_1^{(i)} \cdot \pi\left[L_{e,1}^{(i)}(\hat{u}_k)\right]. \qquad (10)$$

where $\pi[\ ]$ denotes the interleaving operations. Similarly, the SISO2 decoder that the branch, forward, backward path metric, *a-posteriori* information, extrinsic information, Δ function, and scaling factors are calculated as follows

$$\Gamma_{k,2}^{(i)}(s',s) = \frac{1}{2}[x_{k,2}^{s} \cdot L_{a,2}^{(i)}(u_k) + x_{k,2}^{s} \cdot y_{k,2}^{s} + x_{k,2}^{p} \cdot y_{k,2}^{p}], \qquad (11)$$

$$A_{k,2}^{(i)}(s) = \max_{s'}\{A_{k-1,2}^{(i)}(s) + \Gamma_{k,2}^{(i)}(s',s)\}, \qquad (12)$$

$$B_{k-1,2}^{(i)}(s) = \max_{s'}\{B_{k,2}^{(i)}(s) + \Gamma_{k,2}^{(i)}(s',s)\}, \qquad (13)$$

$$L_2^{(i)}(u_k) = \max_{(s',s),u_k=+1}\{A_{k-1,2}^{(i)}(s') + \Gamma_{k,2}^{(i)}(s',s) + B_{k,2}^{(i)}(s)\}$$
$$- \max_{(s',s),u_k=-1}\{A_{k-1,2}^{(i)}(s') + \Gamma_{k,2}^{(i)}(s',s) + B_{k,2}^{(i)}(s)\}, \qquad (14)$$

$$L_{e,2}^{(i)}(\hat{u}_k) = L_2^{(i)}(\hat{u}_k) - y_{k,2}^{s} - L_{a,2}^{(i)}(\hat{u}_k), \qquad (15)$$

$$\Delta_2^{(i)}(\hat{u}_k) = \begin{cases} \frac{1}{N} & if\ \overline{L}_{e,2}^{(i-1)}(\hat{u}_k) \cdot \overline{L}_{e,2}^{(i)}(\hat{u}_k) > 0 \\ 0 & elsewhere \end{cases}, \qquad (16)$$

$$\alpha_2^{(i)} = \sum_{k=1}^{N} \Delta_2^{(i)}(\hat{u}_k) \qquad (17)$$

with the initial condition $\alpha_2^{(0)} = 0.5$. From the extrinsic information $L_{e,2}^{(i)}(u_k)$ of the second SISO decoder, and incorporating the proposed adaptive scaling factor $\alpha_2^{(i)}$ can be derived which is used as the *a priori* information in the next decoder iteration

$$L_{a,1}^{(i+1)}(\hat{u}_k) = \alpha_2^{(i)} \cdot \pi^{-1}\left[L_{e,2}^{(i)}(\hat{u}_k)\right], \qquad (18)$$

where $\pi^{-1}[\ ]$ denote the de-interleaving operations. When the value of $\alpha_2^{(i)}$ is equal to 1.0 then terminates the iterative decoding process at iteration i.

3. SIMULATION RESULTS

In this section, Simulations were carried out with real number Max-Log-MAP (MLM) algorithms, and the encoder specified in 3GPP [5] standard were used. We conduct a series of experiments to evaluate the effectiveness of the adaptive scaling scheme we proposed for turbo decoding. A block of 1024 bits is considered and 50,000 blocks are transmitted. The turbo coding parameters are two 8-state RSC constituent encoders with generator polynomials $(13,15)_{oct}$ linked together by a pseudorandom interleaver [5]. The overall code rate is $1/3$. The coded bits are modulated using binary phase shift keying (BPSK) and white Gaussian noise $n(t)$ with a double-sided power spectral density of $N_0/2$ is added to the modulated signal.

Figure 1 shows the BER plotted against E_b/N_0 for the turbo Max-Log-MAP decoder without scaling (labeled as “MLM”), with fixed scaling (labeled as “FIXSFMLM”), with Max-Log-MAP with LDA criterion without scaling (labeled as “LDASPMLM”), and the proposed scheme with adaptive scaling (labeled as “LDA SFMLM”). In this simulation, the number of decoding iterations is fixed and each block is decoded for 8 maximum iterations. Unlike the “FIXSFMLM” case using a set of pre-determined scaling factors defined in equation (1), the “LDASFMLM” case adaptively computes the scaling factors $\alpha_1^{(i)}$ and $\alpha_2^{(i)}$ according to the equations (9) and (17) on a block by block basis. As can be seen the performance difference between the case “LDASFMLM” and the case “MLM” is about ~0.15 dB to 0.2 dB for the range of E_b/N_0 values simulated. The performance of the case “LDASFMLM” is more superior to that of the case “FIXSFMLM”. At a BER of 10^{-5} the performance difference between the fixed and adaptive scaling schemes is about ~0.1 to 0.2 dB.

Figure 2 shows the FER plotted against E_b/N_0 for a frame/block error rate (FER) of $\sim 10^{-3}$ the improvement of the LDASPMLM is ~0.2dB than the other cases.

Figure 3 shows the average number of iterations plotted against E_b/N_0. From this figure, we can observe that the FIXSFMLM scaling scheme is not effective to prevent unnecessary computation and decoding delays. This is because that its scaling scheme is not offered a stopping rule. And also indicates the proposed adaptive scaling scheme, LDASFMLM, can be embedded with a stopping criterion to saver roughly $1/4 \sim 1/2$ iteration on average than the other cases with additional improvement in BER, FER performances.

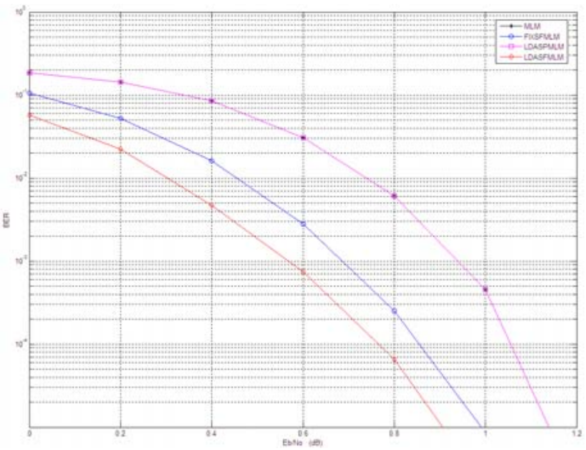

Fig. 1. Comparison of BER performance with different scaling factors and 8 maximum iterations.

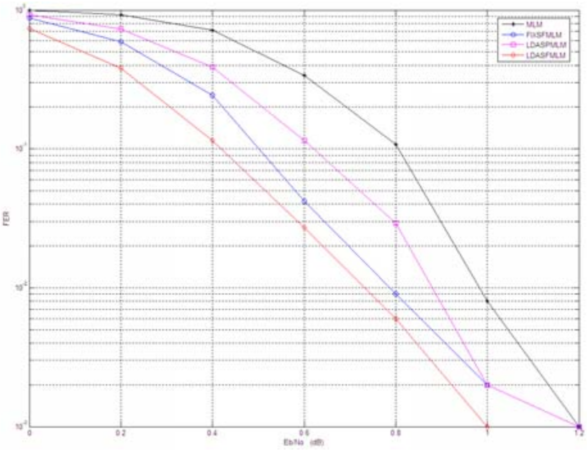

Fig. 2. Comparison of FER performance with different scaling factors and 8 maximum iterations.

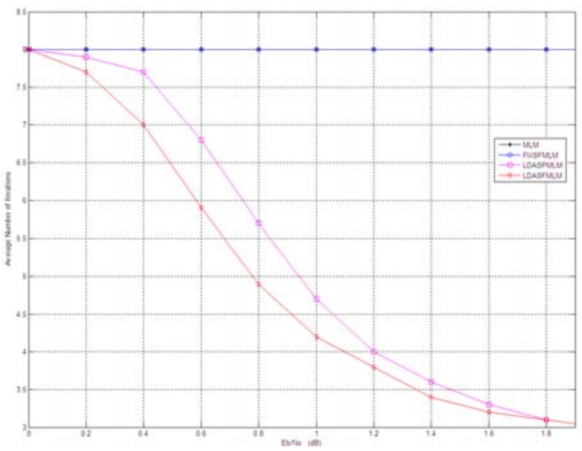

Fig. 3. Average number of iterations versus E_b/N_0 with different scaling factors and 8 maximum iterations.

4. Conclusions

In this paper, a new scaling factor method for improving the Max-Log-MAP algorithm for convolutional turbo decoders has been presented. Applying an efficient scaling scheme and compared it with the existing scaling scheme. The idea of our approach is to first check the sign consistency in log-likelihood-ratio (LLR) information between two consecutive iteration $(i-1)$ and i, and then adaptively determines the corresponding scaling factor for each data block. It has been shown by simulations that the LDA-based scaling technique achieves better performance in terms of BER, FER and the average number of iterations than the other scaling techniques we discussed.

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