A Constellation Extension Based BCM-SLM Scheme for PAPR Reduction of OFDM Signals

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Abstract— **The constellation extension scheme (CES) and selective mapping (SLM) are two of multiple signal representation (MSR) techniques used in OFDM systems for PAPR reduction. The MSR techniques is to generate a number of candidate signals representing the input data and select one with minimum PAPR as the transmitted OFDM signal from these candidate signals. Although CES and SLM are the two efficient PAPR reduction methods, they don't have inherent error correction capability to counteract various channel noise. Therefore, this paper proposes a novel PAPR reduction method with error correction capability used in a 16-QAM OFDM system by combining SLM, CES, and 16-QAM block coded modulation (BCM) codes. The proposed method, called CES-BCM-SLM, does not need to send additional bits of side information and the transmission signal possesses error correction capabilities to prevent interference.**

Keywords—**OFDM, PAPR, SLM, CES, BCM**

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a multi-carrier modulation techniques which has the merit of efficiency on frequency spectrum, robustness to multi-path fading, etc. Consequently, it has been adopted by many communication standards such as 3GPP long term evolution (3GPP LTE) technology, the broadband wireless access standard HIPERMAN and IEEE 802.16x etc [1], [2]. In OFDM systems, each transmitted signal is composed of a large number of independent subcarriers. However, high peak-to-average power ratio (PAPR) of transmitted OFDM signals is major drawbacks of OFDM systems. It will cause signal distortion because of the nonlinearity of high power amplifier (HPA). Thus, PAPR reduction has since become a popular issue. There are many schemes available to reduce the PAPR in OFDM such as clipping [3], coding [4], tone reservation (TR) [5], tone injection (TI) [5], and active constellation extension (ACE) [6]. Especially, selective mapping (SLM) [7] and partial transmit sequence (PTS) [8] are both attractively probabilistic schemes for PAPR reduction, generating a set of candidate signals and select the one with the lowest PAPR for transmission. These schemes, while having have good PAPR reduction performance, also have higher hardware complexity and input must send side information for receiver reduction data [9], [10].

Constellation extension scheme (CES) have been developed for PAPR reduction without transmitting the side information, which are yet another of PAPR reduction scheme appeared recently, often introduced to combine with SLM or PTS technique, which maps the symbols to the points in the original constellation or points in the extended constellation. In [11], a constellation extension algorithm whose reduction performance is almost close to the theoretical limit when the number of sub-carrier is large. In this paper, a new CES BCM-SLM architecture called CES-BCM-SLM is proposed, and is applied to 16-QAM OFDM modulation system, to avoid trans-mission SI while enabling error correction. Compared to previous related CE schemes, simulation results show that the proposed scheme has better performance in reducing PAPR performance and error correction capability than the other PAPR reduction methods.

2. BACKGROUND

2.1. OFDM Signals and PAPR

An OFDM signal is the sum of an input symbol sequence *X*, Let $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]$ be an input data block with N symbol. The continuous sine wave for modulation is defined as $\varphi_n(t) = e^{(j2\pi nt/t)}, n = 0, 1, \ldots, N-1$, For each OFDM transmission signal $x(t)$ with N subcarriers can be written as :

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \phi_n(t), 0 \le t \le T , \quad (1)$$

where 1/T is the bandwidth of each subcarriers. The instantaneous envelope power $P_x(t)$ of signal $x(t)$ can be written as $P_x(t) = |x(t)|^2$, where PAPR of $x(t)$ is defined as:

$$\mathrm{PAPR}(x(t)) = \frac{\max_{0 \le t \le T} P_x(t)}{P_{av}}, \quad (2)$$

where P_{av} is the average power of $x(t)$. Generally, for accurate approximation of the PAPR in signal $x(t)$, discrete LN-1 and (N-1) zero-padding for IFFT calculation. In this paper, L=4 has been used for numerical simulation.

2.3. Conventional SLM Schemes

Fig. 1 shows the block diagram of the conventional SLM scheme. The conventional SLM scheme is based on multiplying the input data block by M unit amplitude scrambling sequences $y_i = \{e^{j\phi_{i,0}}, \cdots, e^{j\phi_{i,n-1}}\}$, $1 \le i \le \mathrm{M}$, to form M distinct statistically independent candidate sequences, representing the same input block. The phase $\phi_{i,j}$ is chosen as 0 or π . The transmitted sequence is then selected with minimum PAPR from candidate sequence. To recover the input data block in the receiver, the number of $\lceil \log_2 M \rceil$ bits, called side information, should be transmitted reliably at the transmitter. The performance of PAPR reduction in SLM strongly depends on the number and selection of scrambling sequences.

2.4. Constellation Extension Schemes

Fig. 2 shows a 16-QAM signal constellation and the CES using the Gray Mapping, Fig. 2(a) is a 16-QAM signal constellation and each symbol maps to on input data. The 16-QAM constellation extension scheme is part of symbols with two mapping input data. According to input data, 16-QAM constellation extension scheme decimal values is greater than or equal to 4, the amount of mapped symbols will increase, as show in Figure. 2(b). For example, when the input data is (1,1,1,1)

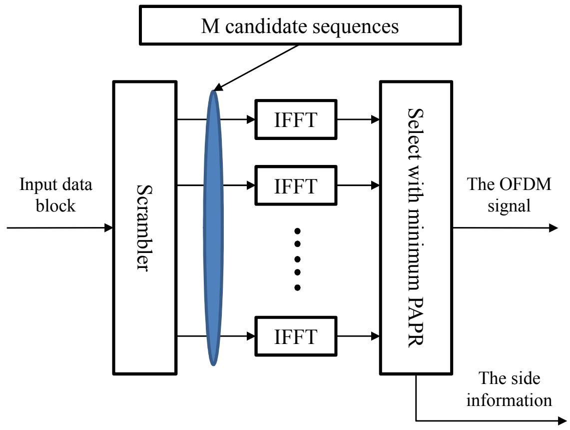
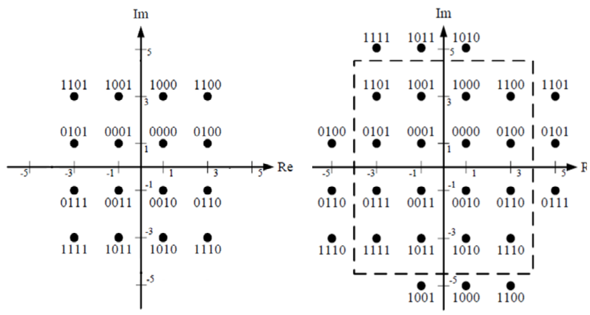


Fig.1 The block diagram of the SLM approach



(a) Gray mapping of the 16-QAM signal constellation

(b) Gray mapping of the 16-QAM constellation extension scheme

Fig. 2 Extension 0f 16-QAM constellation for (a) and (b)

then the bit mapping based on contents of the symbol in addition to some -3-3j, but also has an extended symbol -3+5j. On the other hand, because CES uses a modulation method to mapping the input data into two symbols, the receiver can demodulate the signal to obtain the sent input data. Therefore, to recover the input data in the receiver, the transmitter has no need to transmit the side information.

2.5. BCM-SLM Schemes

The BCM-SLM algorithm is a combination of two approaches to improve the performance of a BCM-OFDM system. A BCM achieves bandwidth efficient modulation with good Euclidean distance, while SLM improves the PAPR performance. Let C_1, C_2, C_3 and C_4 each be a binary code of length n with minimum Hamming distances d_i, $1 \le i \le 4$, respectively. A $4 \times n$ codeword array is constructed by filling the ith row with codewords form the ith component code C_i as follows :

$$\begin{pmatrix} c_1 \in C_1 \\ c_2 \in C_2 \\ c_3 \in C_3 \\ c_4 \in C_4 \end{pmatrix} = \begin{pmatrix} c_{1,0} & c_{1,1} & \cdots & c_{1,n-1} \\ c_{2,0} & c_{2,1} & \cdots & c_{2,n-1} \\ c_{3,0} & c_{3,1} & \cdots & c_{3,n-1} \\ c_{4,0} & c_{4,1} & \cdots & c_{4,n-1} \end{pmatrix}, \tag{3}$$

Then, a complex-valued sequence of length n in a 16-QAM BCM code B can be formed by modulating each column of the $4 \times n$ codeword array into a 16-QAM symbol. The 16-QAM signal constellation is shown in Fig. 3, with bit labels assigned by a set partitioning scheme in which the intra-subset squared Euclidean distances rise in the ratio 1:2:4:8. More precisely, a 16-QAM BCM codeword $b = [b_0, b_1, \ldots, b_{n-1}] \in B$ can be written as :

$$b_k = \left(\frac{1}{\sqrt{2}} j^{c_{1,k}+2c_{2,k}} + \sqrt{2} j^{c_{3,k}+2c_{4,k}} \right) e^{j\pi/4}, \tag{4}$$

$0 \le k \le n-1$, in terms of these four binary component codewords $c_i \in C_i$.

The minimum squared Euclidean distance between any two codewords in B is known as

$$D_E^2 = \min_{1 \le i \le 4} d_i \cdot 2^{i-1} \cdot E^2, \tag{5}$$

where E is the minimum Euclidean distance between signal points. To increase the code rate and balance the minimum squared Euclidean distance of a resulting 16-QAM BCM code, we require $d_1 = 2d_2 = 4d_3 = 8d_4$. For example, the second order Reed-Muller code is generated by $RM(2,m) = \langle 1, x_1, \cdots, x_m, x_1x_2, \cdots, x_{m-1}x_m \rangle$.

A generator matrix G of a 16-QAM BCM code can be written as :

$$G = \begin{pmatrix} G_1 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 \\ 0 & 0 & G_3 & 0 \\ 0 & 0 & 0 & G_4 \end{pmatrix}, \tag{6}$$

where G_1, G_2, G_3 and G_4 are the generator matrices of the component C_1, C_2, C_3 and C_4, respectively. The algorithm of BCM-SLM can be described as follows.

1) The subcode of the order Reed-Muller code except the zero order Reed-Muller code, $\mathrm{RM}(1,m)/\mathrm{RM}(0,m) = \langle G_Y \rangle$ is chosen as the encoder of PAPR control bits.

2) The input bits of $G_4 = G_X \cup G_Y$ is divided into information bits with generator matrix, G_X and PAPR control bits with generator matrix, G_Y.

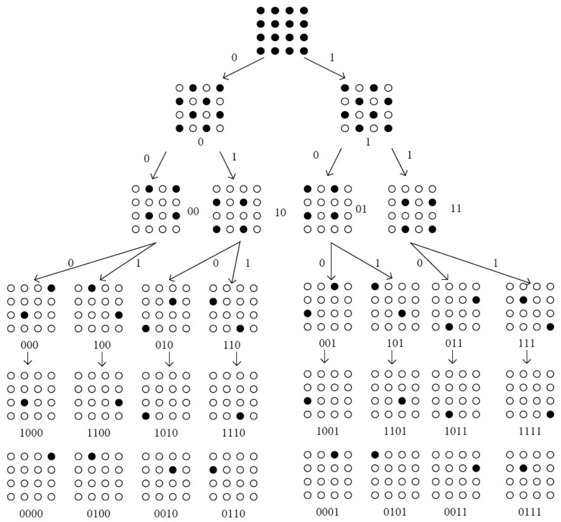


Fig. 3 Set partitioning for 16-QAM constellation

3) After encoding the PAPR control bits, binary scrambling sequences from the PAPR subcode $\langle G_Y \rangle$, can be acquired, which is added with the codeword generated by the generator matrix G_X and then use these resulting sequences with the codewords from the component code C_1, C_2, C_3 and C_4 to form a number of $4 \times n$ codeword arrays.

4) Finally the 16-QAM candidate sequences can be acquired by mapping these codeword arrays into the 16-QAM sequences and the one with minimum peak power is transmitted.

3. The Proposed Method and Simulation Results

The proposed extension scheme combined with BCM-SLM can effectively reduce PAPR. A generator matrix G of 16-QAM BCM codes is composed of four generator matrices. Let G_1, G_2, G_3 and G_4 are the generator matrices of the component code C_1, C_2, C_3 and C_4, respectively. The algorithm of CE-BCM-SLM can be described as follows:

1) The choose subcode of the first order Reed-Muller code except the zero order Reed-Muller code, $\mathrm{RM}(1,m)/\mathrm{RM}(0,m) = \langle G_Y \rangle$.

2) The input bits of $G_4 = G_X \cup G_Y$ into the information bits with generator matrix G_X and matrix G_Y , G_Y of the subcode to determine internal point and external point for 16-QAM constellation points.

3) The proposed 16-QAM extended constellation is shown in Fig. 4, where it has 16 extendable constellation points.16QAM constellation points is divided into extends a different distance from the 7 case, accordance with each case. Case 1 conditional extension distance with four changes, $\pm 1 \pm j2$ and $\pm 2 \pm j$, respectively. Case 2 to case 7 separately extend distance according to the conditional formula ± 4 and $\pm 4j$ are eight changes.

The above seven case use adder instead complexity higher of the multiplier, while is extends inside constellation extends to the external extension constellation. Extending 7 constellation points, as shown in Table 1, can be described by 2 cases as follows :

Case 1 : when $c_2 = \bar{c}_4$ and $c_1 = c_3$, among $c_2 = 1$, if $c_1 = \bar{c}_2$ then (0.5,0.5) points extend add 1+j2, extends after constellation is (1.5,2.5), if $c_1 = c_2$ then (-0.5,0.5) point extend add -2+*j*, extends after constellation is (-2.5,1.5); among when $c_2 = 0$, if $c_1 = \bar{c}_2$ then (-0.5,-0.5) point extend add -1-2*j*, extends after constellation is (-1.5,-2.5), if $c_1 = c_2$ then (0.5,-0.5) point extend add 2-*j*, extend after constellation is (2.5,-1.5).

Case 2: when $c_2 = c_4 = 0$ and $c_1 = \bar{c}_3$, then if $c_1 = 0$ or $c_1 = 1$, symbol point (-0.5,1.5) or (0.5,1.5) extend add –4*j*, extend after constellation represent the (-0.5,-2.5) or (0.5,-2.5). it is respective the {0100} or {0001} symbol data. Note, assuming extends inside the extends outside constellation point (-0.5,1.5) and (-0.5,-2.5), they are representation {0100} symbol data. Where $\bar{v}$ is the bit invert, *j* express imaginary, according to the above description, the binary description of the 16-QAM constellation point can be represented as ($c_{4,k}$, $c_{3,k}$, $c_{2,k}$, $c_{1,k}$), $1 \le k \le n$, among $c_{1,k}$ is the least significant bit and $c_{4,k}$ is the most significant bit.

Complementary cumulative distribution function (CCDF) curve is adopted to evaluate the PAPR reduction performance for different SLM scheme. The CCDF can be expressed as the probability that PAPR, exceeds a presented PAPR_0 as show in following:

$$CCDF(PAPR_0) = \Pr\left(PAPR\{x\} > PAPR_0\right). \qquad (7)$$

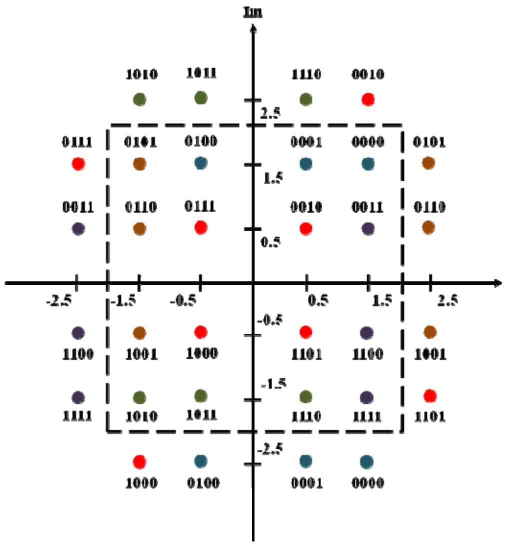


Fig.4 The proposed extendable constellation

The PAPR reduction performance of the proposed scheme in Fig. 4 is investigated using computer simulations. During the simulations, it is assume that the OFDM system has 256 subcarriers, where *L*=4 is adopted to approximate the true PAPR. The amount of candidate signals is denoted by *M*. For comparison, [11] is considered in the simulations.

The conventional SLM [7] is also revealed, where the phase rotation vectors adopted are randomly drawn with -1 and 1. [11] and proposed with constellation extension scheme, all use original 16-QAM extend to 32-QAM, they are extend 16 constellation point. Proposed adopt natural order Reed-Muller code, use

$$\{RM(1,\ \log_2 N),\ RM(2, \log_2 N),\ RM(3,\ \log_2 N),\ RM(4,\ \log_2 N)\}$$

and

$$\{RM(2,\ \log_2 N),\ RM(3,\ \log_2 N),\ RM(4,\ \log_2 N),\ RM(5,\ \log_2 N)\}$$

produce 16-QAM codeword. The minimum square Euclidean distances of these resulting 16-QAM codes is 2^{m-2}.

Fig.5 can be observed, when *N*=256, *M*=8, the selection of different Reed-Muller code to decision the 16-QAM is extending to the of constellation points, wherein *RM*(4,8) reduce PAPR, compared to selection of *RM*(3,8), *RM*(2,8), *RM*(1,8) performance to better. For the C-SLM and [11], proposed scheme using the *RM*(4,8) to be able to reduce 0.2dB and 0.15dB. One the other hand, when *N*=256, *M*=8, we increase the amount of the order of the RM code, while can reduce the PAPR performance with minor slightly enhance 0.1dB compared with a

TABLE 1

PROPOSED EXTENDABLE CONSTELLATION SCHEME CASE TABLE

Case	Conditions	Extended Distances
Case 1	$\{c_2=\bar{c}_4, c_1=c_3\}, \{c_2=1, c_1=\bar{c}_2\}$	1+2*j*
	$\{c_2=\bar{c}_4, c_1=c_3\}, \{c_2=1, c_1=c_2\}$	-2+*j*
	$\{c_2=\bar{c}_4, c_1=c_3\}, \{c_2=0, c_1=c_2\}$	-1-2*j*
	$\{c_2=\bar{c}_4, c_1=c_3\}, \{c_2=0, c_1=\bar{c}_2\}$	2-*j*
Case 2	$\{c_2=0, c_2=c_4\}, \{c_1=\bar{c}_3\}$	$-4j$
Case 3	$\{c_2=1, c_2=c_4\}, \{c_1=\bar{c}_3\}$	$4j$
Case 4	$\{c_1=c_2, c_3=c_4\}, \{c_1=\bar{c}_3\}$	-4
Case 5	$\{c_1=\bar{c}_3, c_2=\bar{c}_4\}, \{c_1=c_3\}$	4
Case 6	$\{c_1=c_3, c_2=c_4\}, \{c_1=0\}$	$4j$
	$\{c_1=c_3, c_2=c_4\}, \{c_1=1\}$	4
Case 7	$\{c_1=c_2=c_3=c_4\}, \{c_1=0\}$	$-4j$
	$\{c_1=c_2=c_3=c_4\}, \{c_1=1\}$	4

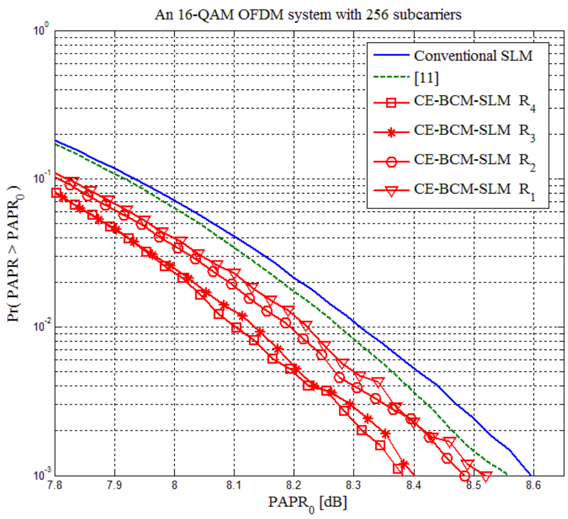


Fig. 5 The CCDFs of PAPR in CE-BCM-SLM with $(c_{4,k}, c_{3,k}, c_{2,k}, c_{1,k})$ = { $RM(1,8)$, $RM(2,8)$, $RM(3,8)$, $RM(4,8)$ }.

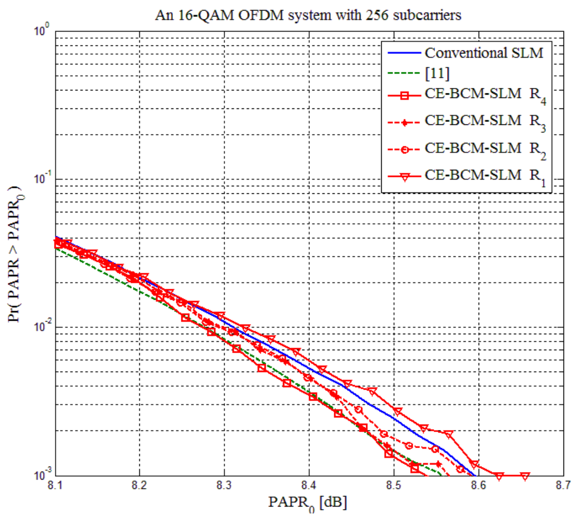


Fig. 6 The CCDFs of PAPR in CE-BCM-SLM with $(c_{4,k}, c_{3,k}, c_{2,k}, c_{1,k})$ = { $RM(2,8)$, $RM(3,8)$, $RM(4,8)$, $RM(5,8)$ }.

{ $RM(1, \log_2 N)$, $RM(2, \log_2 N)$, $RM(3, \log_2 N)$ and $RM(4, \log_2 N)$} from Fig.6. It is worth mentioning that, although the order to enhance the *RM* code amount will increase PAPR, but the error correction capability relatively low amount of order to the better.

Note that, conventional SLM has to send the side information to the receiver for data recovery. On the contrary, the proposed scheme and [11] avoids side information, but proposed scheme compared subjected to noise interference, proposed scheme has better BER performance and error correction.

4. CONCLUSIONS

This paper proposes a modified CES based on the architecture of 16-QAM BCM-SLM scheme, called CES-BCM-SLM. The CES-BCM-SLM does not need to transmit the side information in the transmitter. Moreover, each transmitted signal of CES-BCM-SLM scheme has error correction capability to counteract various channel noise. Simulation results show that our proposed method has better PAPR performance than the other two PAPR reduction methods.

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