

Gravitational Search Algorithm for Low Complexity Carrier Frequency Offset Estimation in Uplink OFDMA Systems

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Abstract—Orthogonal frequency division multiple access (OFDMA) systems in the uplink suffer from multiple access interference (MAI) due to their high sensitivity to frequency misalignments between different users. Moreover, carrier frequency offset (CFO) is a major factor that degrades performance of the OFDMA system. As the maximum-likelihood (ML) estimator is impractical in this context, we introduce a family of suboptimal estimators with the aim of exhibiting an attractive tradeoff between performance and complexity. The complexity reduction in the proposed solutions is substantial when compared to the existing ones in the literature. It is worth mentioning that this substantial complexity reduction is achieved in expense of some bandwidth efficiency loss. Hence, there exists a tradeoff between bandwidth efficiency and complexity. In this paper, we propose a new approach called gravitational search algorithm (GSA), for CFO estimation. Performance and complexity comparisons are provided, along with numerical results to illustrate the effectiveness of the proposed method. Simulation results demonstrating the effectiveness of the proposed algorithms in approaching the optimal performance are also presented.

Keywords- OFDMA; gravitational search algorithm; carrier frequency offset, ML

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is an ideal multi-carrier multiple access technique for high data rate wireless communication systems because of its high spectral efficiency, flexibility for quality of service (QoS) support and robustness to multipath fading effects. In OFDMA uplink transmissions, there are two challenging issues. However, in high data rate wireless systems employing OFDMA, such as Long Term Evolution-Advanced (LTE-Advanced) based cellular wireless networks and IEEE 802.16m based worldwide interoperability for microwave access (WiMAX) with highly mobile users, the transmitted signal quite often undergoes both time and frequency selective (doubly selective) fading due to high Doppler spread and multipath effects. The first one is frequency synchronization since OFDMA, which is an orthogonal frequency-division multiplexing (OFDM)-based technique, is particularly sensitive to carrier frequency offsets (CFOs). Moreover, in an OFDMA uplink, different users introduce different CFOs; therefore, the CFO estimation at the base station is a

multivariable estimation problem, which is computationally expensive. The second challenge is channel estimation since a large number of channel parameters need to be estimated for coherent detection. It is well known that OFDMA inherits from OFDM the weakness of being sensitive to carrier frequency offset (CFO) that is generated by the frequency mismatch between the transceiver oscillators or the Doppler effect. Nevertheless, to support access for a large number of users, the number of subcarriers, N , in OFDMA systems is usually very large, which causes a high complexity. Moreover, OFDMA [3-4], the presence of CFO between the uplink signals and the base station (BS) local reference gives rise to interchannel interference (ICI) as well as multiple-access interference (MAI), with ensuing limitations of the system performance. In particular, the algorithms derived in [5] and [6] are based on the maximum likelihood (ML) principle and provide CFO estimates by exploiting a training block transmitted by each user at the beginning of the uplink frame. In spite of their effectiveness, such solutions are too computationally demanding as they require a complete search in order to locate the maximum of the likelihood function. Some computational saving is achieved in [7] by replacing the exhaustive search with a line search. This results into a scheme with reduced complexity but slower convergence rate. A high-performance ML algorithm for both synchronization and channel estimation for an OFDMA uplink transmission is studied in [7], and the complexity is reduced by employing an alternating projection method. On the other hand, the approximation should be fine enough so that the estimation remains (almost) optimal in the ML sense. Thus, our aim is to define a simplified estimation algorithm whose performance becomes identical to the performance of the exact ML algorithm when the number of subcarriers increases. An iterative time and frequency synchronization scheme using the space-alternating generalized expectation-maximization algorithm for interleaved OFDMA uplink systems is proposed in [4]. The MAI cancellation in an OFDMA system is discussed in [8]. A conventional estimator, such as in [9], is considered as a candidate for frequency offset estimation in [8].

The effect of the CFOs on multicarrier systems was extensively investigated in the literature [10–13]. In these detectors, error floors, which are mainly induced by the

large MAI, appear. In [11], a circular convolution detector is used to estimate the MAI in the frequency domain. Then, parallel interference cancellation (PIC) is used to regenerate and remove the MAI from the original signal. To overcome the difficulty of correlation-based methods, ML-based approach has been explored in recent years [12-13]. ML-based scheme searches all possible solutions in the solution space and chooses the best one as final result. Theoretically, ML-based approach can find the optimal solution and has the immunity against the amount of CFO. Unfortunately, its complexity is too high for practical application; especially when the solution space is multi-dimensional.

One of the most new evolutionary algorithms is gravitational search algorithm (GSA) which has been presented by Rashedi et al. [15] in 2009. GSA is applied in optimisation problem with different objective functions in [15] and obtained results are compared with particle swarm optimisation (PSO) and residual gas analyser (RGA). It has been demonstrated that obtained results of GSA are better than those obtained by PSO and RGA algorithms [15]. The GSA concept is simple and known as a powerful optimisation algorithm; besides significant privileges of this algorithm it has some drawbacks too, such as it might be trapped in local optima in some cases. Unlike ML approach exhaustively finding all possible solutions in the solution space, GSA is a kind of evolutionary computation techniques like PSO. It can estimate all CFOs at a time. GSA randomly generates some particles (candidate solutions), which fly in the space to find the best solution according to individual and whole experiences. In this work, a low-complexity algorithm for CFO recovery in the OFDMA uplink is proposed. This paper is organized as follows. The signal model of OFDMA is presented in Section II. The periodic signal structure of the interleaved OFDMA uplink is introduced in Section III. In Section IV, the CFO-estimation problem is stated and the proposed estimation algorithm is derived. Simulation results are reported in Section V, and conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider an OFDMA uplink system with N subcarriers and K active users. In this paper, we assume that the carrier frequency offsets of uplink users are known, or estimated, by the receiver. After the bitwise interleaving, they are mapped into quadrature amplitude modulation (QAM) symbols. The number of subcarriers that is assigned to each user can be given as $M = N/K$. The subcarriers include all available subcarriers and virtual subcarriers in the guard band [11-13]. Before initiating the transmission, the timing for each user is acquired by using the downlink synchronization channel from the base station. Consequently, the transmissions from all users can be regarded as quasi-synchronous. The total number of subcarriers is denoted as N , and one block of frequency-

domain symbols that are sent by the k th user is denoted as $d_k = [d_k(0), d_k(1), \dots, d_k(N-1)]^T$, where $d_k(i)$, $i = 0, \dots, N-1$, is nonzero if and only if the i th subcarrier is modulated by the k th user. For virtual subcarriers, the modulation symbols are effectively padded zeros in IFFT. For pilot subcarriers, the modulation symbols are pilot symbols or training symbols for estimating the channels.

Denoting the timing offset that is caused by propagation delay as η_k , the compound channel response can be written as $\mathbf{h}_k = [0_{\eta_k \times 1}^T \mathcal{E}_k^T O_{(L-\eta_k-L_k) \times 1}^T]^T$ where L is the upper bound on the compound channel length. For user K , the normalized carrier frequency offsets (CFO) and the phase offset (between the oscillator at user k and that of the base station) are denoted as ε_k and θ_k , respectively. At the base station, after timing synchronization and removal of CP, the signal from user k is given by

$$\mathbf{x}_k = [x_k(0), x_k(1), \dots, x_k(N-1)]^T = \Gamma(w_k) \mathbf{A}_k (\exp(j\theta_k) \mathbf{h}_k) \quad (1)$$

$$\text{where } \Gamma(w_k) = \text{diag}(1, \dots, \exp(j(N-1)w_k)) \quad ,$$

and $\mathbf{A}_k = F^H \mathbf{D}_k F_L$, and $w_k = 2\pi\varepsilon_k / N$. In the above signal model, the phase offset $\exp(j\theta_k)$ can be incorporated into the channel response \mathbf{h}_k . This renders the estimation of the phase offset dispensable since only the combined channel $\exp(j\theta_k) \mathbf{h}_k$ is needed in the equalization.

We further define θ_k as the *effective CFO* of the k th user. As will be shown later, the proposed algorithm estimates the effective CFO rather than the normalized CFO directly. Effective CFO has one important property. Different users have distinct effective CFOs. Based on this fact, the CFO estimation will not be influenced by the presence of the common phase offset. Thus, without loss of generality, we assume that $\theta_k = 0$, $\forall k$, for simplicity. Since the received signal at base station \mathbf{x} is a superposition of the signals from all the users plus noise, we have

$$\mathbf{x} = \sum \Gamma(w_k) \mathbf{A}_k \mathbf{h}_k + \mathbf{n} \quad (2)$$

$$\mathbf{n} = [n(0), n(1), \dots, n(N-1)]^T \quad (3)$$

is the complex white Gaussian noise vector with zero mean and covariance matrix $G_n = E\{\mathbf{n}\mathbf{n}^H\} = \sigma^2 \mathbf{I}_{N \times N}$. The discrete-time channel impulse response of the k th user is $h = [h_1^T \dots h_k^T]^T$, signal model (2) can be rewritten as

$$\mathbf{x} = \mathbf{Q}(w) \mathbf{h} + \mathbf{n} \quad (4)$$

where $\mathbf{Q}(w) = [\Gamma(w_1) \mathbf{A}_1 \Gamma(w_2) \mathbf{A}_2 \dots \Gamma(w_K) \mathbf{A}_K]$.

In a multiple-access system, the unknown parameters of interfering users can be treated as nuisance parameters, which will degrade the estimation accuracy of the parameters of interest. Regarding OFDMA uplink frequency offset estimation, the following result holds. On the other hand, [7] proposes a maximum-likelihood (ML)-based

alternating-projection algorithm.. However, the algorithm of [7] is still computationally demanding. For instance, the algorithm is difficult to implement in situations where the number of subcarriers is large. It is, therefore, of practical interest to propose suboptimal estimators which are likely to be implemented in such contexts, i.e., which have reasonable complexity and, on the other hand, which have a performance close to the ML performance. Based on the signal model in (4), the ML estimate [7] [11-14] of parameters $\{\mathbf{h}, \mathbf{w}\}$ is given by maximizing

$$\Psi(\mathbf{x}, \hat{\mathbf{h}}, \hat{\mathbf{w}}) = \frac{1}{(\pi\sigma^2)^N} \exp\left\{-\frac{1}{\sigma^2} [\mathbf{x} - \mathbf{Q}(\hat{\mathbf{w}})\hat{\mathbf{h}}]^H [\mathbf{x} - \mathbf{Q}(\hat{\mathbf{w}})\hat{\mathbf{h}}]\right\} \quad (5)$$

where $\hat{\mathbf{h}}$ and $\hat{\mathbf{w}}$ are trial values of \mathbf{h} and \mathbf{w} can be obtained as

$$\hat{\mathbf{w}} = \arg \max_{\hat{\mathbf{w}}} \left\{ \Lambda(\hat{\mathbf{w}}) = \|\Lambda_{\mathbf{Q}}(\hat{\mathbf{w}})\mathbf{x}\|^2 \right\} \quad (6)$$

where $\Lambda_{\mathbf{Q}}(\hat{\mathbf{w}}) = \mathbf{Q}(\hat{\mathbf{w}})(\mathbf{Q}^H(\hat{\mathbf{w}})\mathbf{Q}(\hat{\mathbf{w}}))^{-1}\mathbf{Q}^H(\hat{\mathbf{w}})$ [7]. The CFO estimation in (6) requires an exhaustive search over the multidimensional space spanned by $\hat{\mathbf{w}}$, which may be too computationally expensive in implementation. Moreover, the maximization problem (6) can be directly solved by using an exhaustive grid-search over the multi-dimensional space spanned by $\hat{\mathbf{w}}$, which requires prohibitively large computational complexity and is clearly not feasible in practice. The resulting procedure consists of *cycles* and *steps*. A cycle is made of K steps, and each step updates the CFO of a single user while keeping the other CFOs constant at their most updated values. Without loss of generality, we follow the natural ordering $k=1, 2, \dots, K$ in updating the users' CFOs. Also, we denote \hat{w}_k^j the estimate of w_k at the i th cycle and define the $(K-1)$ -dimensional vector as

$$\hat{\mathbf{w}}_k^j = [\hat{w}_1^{j+1}, \dots, \hat{w}_{k-1}^{j+1}, \hat{w}_{k+1}^j, \dots, \hat{w}_K^j]^T \quad (7)$$

III. EASE OF USE ESTIMATION SCHEME USING GRAVITATIONAL SEARCH ALGORITHM (GSA)

TRashedi et al. proposed a new meta-heuristic searching algorithm called GSA in 2009. This algorithm is introduced based on Newton's law of gravity and law of motion. The law of gravity states that every particle attracts every other particle and the gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. According to the proposed algorithm, agents are assumed to be objects, the performances of which are measured by means of masses [15-19]. The GSA is established on the law of gravity and the concept of mass interactions. It uses the theory of Newtonian physics and its searcher agents are the collection of masses. In physics, gravitation is a tendency in which objects with mass accelerate towards each other. In Newton gravitational law, each particle attracts the other particle with a force called the 'gravitational force' [16-18].

GSA uses objective functions as fitness performance assessment, with position of each of these particles as a solution, and the masses adjusted according to their performance at each iteration. To describe the presented algorithm, we assume a system with n masses in which the location of i th mass is described as follows

Considering N particles (agents) and a q -dimensional search space, the position of i th agent is defined by the vector X_i [17]

$$X_i = [x_i^1, x_i^2, \dots, x_i^l, \dots, x_i^q], i = 1, 2, \dots, N \quad (8)$$

Where x_i^l is the position of l^{th} dimension, $l = 1, 2, \dots, q$. After computing mass for each agent, it is possible to compute acceleration for every single one of them; to this end, total force from a set of heavier masses which is applied on each agent is computed as follows, based on Newton rules. According to the law of motion the acceleration of i^{th} agent in l^{th} dimension defined as follows the index k :

$$y_i^l(k) = \left[\frac{1}{z_{li}}(k) \right] \quad (9)$$

$$\sum_{j=1, j \neq i}^N \rho \left\{ f(k) z_{pi}(k) \cdot z_{Aj} [x_j^l(k) - x_i^l(k)] / r_{ij}(k) + \delta x_i^l(k) \right\}$$

where ρ , $0 \leq \rho \leq 1$, is a random variable. $f(k)$ is the viable of the gravitation constant at the iteration k , $z_{li}(k)$ is the inertia mass of i^{th} agent, $z_{pi}(k)$ is the active gravitation mass of i^{th} agent. $\delta > 0$ is a small constant introduce in order to eliminate a possible division by 0., and $\psi_{ij}(k)$ is the Euclidian distance between i^{th} and j^{th} agent

$$\psi_{ij}(k) = \|\mathbf{x}_i(k) - \mathbf{x}_j(k)\| \quad (10)$$

The distance is used in (9) instead of the square distance to reduce the computational complexity according to [17-19]. As analyzed in [18], this will alter the influence of the distance between particles by reducing the distance size and focusing on the direction of the acting force. In addition, the effect of gravity constant depreciation is introduced. The gravitational constant decrease with the advance of GSA's iterations can be modeled by several depreciation laws, of which we focus on [17]

$$g(k) = g_0(1 - \phi k / k_{\max}) \quad (11)$$

$$g(k) = g_0 \exp(-\xi k / k_{\max}) \quad (12)$$

where k_{\max} is the maximum number of iterations, and g_0

is a preset constant such that to ensure GSA's convergence and to influence the search accuracy, φ and ξ are a priori set parameters which ensure a tradeoff to GSA's convergence and search accuracy, $\varphi > 0$, $\xi > 0$.

The velocity update of i^{th} agent in l^{th} dth dimension, $v_i^l(k+1)$, is considered as a fraction of its current velocity added to its acceleration. Therefore, the position and velocity of an agent are calculated in terms of the following state-space equations

$$v_i^l(k+1) = \beta v_i^l(k) + y_i^l(k) \quad (13)$$

$$x_i^l(k+1) = x_i^l(k) + v_i^l(k+1) \quad (14)$$

where $\beta, 0 \leq \beta \leq 1$, is a uniform random variable. As shown in [20-21], and [22], the above choices of the parameters in the GSA will ensure the global optimum. However, as in the case of other evolutionary nature-inspired optimization algorithms, a reasonable number of iterations will be carried out. In addition, accounting for the random variables specific to GSA, the results will be presented as average values. The gravitational constant decreases during the optimisation process as follows:

Step 1: Define the input data.

Step 2: The objective functions described in (6).

Step 3: An initial population X_i which must meet constraints, is generated randomly.

Step 4: Calculate the objective functions values (6). The augmented objective function (6) is evaluated by using the load flow result. Also, for each individual X_i the membership values of all different objectives are computed.

Step 5: Apply the Pareto method in order to obtain normalised objective function of previous step and save non-dominate solutions in the repository. Compute the weight factor for all non-dominate solutions.

Step 6: Sort all agents according to their weight factors and select the best agent and worst agent among them.

Step 7: Calculate mass for all agents according to (8) and (9).

Step 8: Compute the force which is applied to each agent and its acceleration according to (10). The velocity of each agent is computable having its acceleration according to (11).

Step 9: Update the position of each agent according to (12)

IV. EXPERIMENTAL RESULTS

In this section, we present the BER performance and iteration number of the GSA and GA methods and Differential evolution (DE) algorithm compare them with those of conventional schemes in OFDMA uplink systems with $N = 512$ subcarriers. The conventional CFO estimator is based on the GSA-ML algorithm where the full search algorithm is used to find CFO in each CFO range. The

channel delay spread is less than the length of CP ($N_G = 32$), and CFO is distributed over $[-0.5, 0.5]$. Because different users occupy different subchannels, their effective CFOs fall in non-overlapping ranges. We evaluate the performance of the proposed schemes in terms of the bit error rate (BER) against the signal-to-noise ratio (SNR). Figs. 1 and 2 show the BER versus SNR, where the system parameters with user $K=10$ and 32, respectively. Fig. 1 and 2, we see that the GSA-ML algorithm obtains almost the same results as the simplified ML method does when $\Delta 1$ is large and the SNR is high. Obviously, the estimation errors increase with the increase in the number of users, because more users induce a larger MAI. Due to the MAI, the estimation has an error floor in terms of SNR (the error will reach a fixed value for a large SNR). In Fig. 3, we compare the computational complexity of the two methods in terms of the required average number of multiplications

V. CONCLUSIONS

In this paper, the problem of joint CFO and channel estimation for an OFDMA uplink has been considered, and the ML joint CFO and channel estimator has been derived. In this paper, in order to achieve better trade-off between performance and complexity for the OFDMA uplink system, we proposed a GSA-based ML scheme. The cases of an ideal and practical CFO estimation have been also considered in order to highlights the good behavior for the proposed receiving schemes. The GSA method is very appealing for practical implementation since it achieves almost the same performance as the ML method while requiring significantly lower complexity. Simulations show that the number of clusters over which the GSA-based ML scheme is performed can be chosen adaptively by using the knowledge of frequency offset and BER values for better performance and complexity trade-off. Simulations showed that all solutions lead to the same bit error rate performance and thus the choice between different CFO algorithms is dictated by their computational complexity. The research presented in this paper led to CFO compensation algorithms whose complexity was only one order of magnitude greater than that of the single user case while keeping very close to the optimal performance. Such a substantial complexity reduction makes our algorithms feasible in practice and therefore, suitable for hardware implementation of real-time uplink OFDMA systems.

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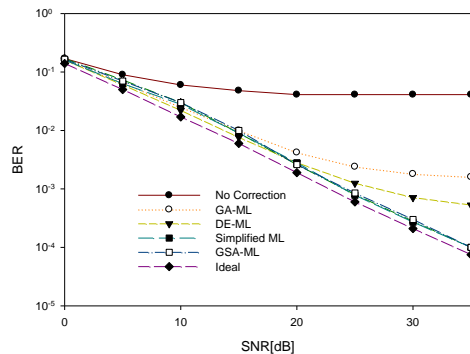


Fig. 1. BERs versus SNRs ($K=10$, GSA-, GA-, DE-ML, simplified ML).

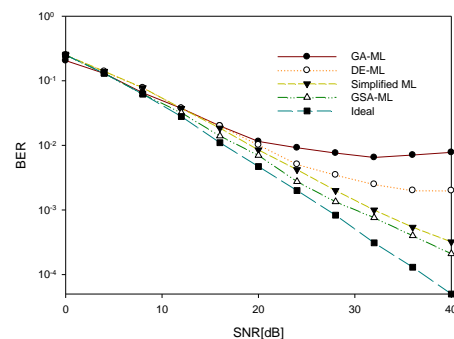


Fig. 2. BERs versus SNRs ($K=32$, GSA-, GA-, DE-ML, simplified ML).